REVIEW

ELECTRIC CONVECTION

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1. Introduction: Lomonosov's paper "A Note on Atmospheric Phenomena Due to Electrical Forces" was published in 1753 in St. Petersburg. The title suggests that the paper examines aerodynamic effects stemming from electrical causes. In actual fact the paper contains the very first correct description and true explanation of the phenomena of thermal convection in general, and in meteorology in particular. In it, thunderstorm phenomena were correctly presented as the effects of ascending streams of warm air.

It is difficult to explain what caused the author to exchange cause and effect in the title. It is, of course, quite clear that the process of thunderstorm generation, not fully elucidated at that time, seemed quite confused to Lomonosov, and the reasoning applied to it contained much that was naive and conjectural.

The idea expressed in the title of this publication may be compared with related problems in science in contemporary conditions. Combination of the ideas of hydrodynamics with the law of conservation of energy has formed a highly elaborate theory of thermal convection. Combination of the ideas of hydrodynamics with the law of conservation of momentum has entered into the constitution of hydrodynamics itself, as the fundamental section on hydrodynamic resistance. Combination of the ideas of hydrodynamics with the law of mass conservation has laid the foundation of a detailed, elaborate study of convective diffusion and of the hydrodynamics of physical chemistry, among other things. Finally, combination of the ideas of hydrodynamics with the law of conservation of electric charge, unilaterally formulated in the title of Lomonosov's work, has not only been developed very little on the scientific plane, but has not even yet been registered as a scientific discipline.

A matter of note is the great attention now riveted to one particular, prominent, and vigorously developing branch of science in this field, namely, magnetohydrodynamics. The concern here is not so much with electric charge, as with electric currents of high density in a plasma with high electrical conductivity. One might mention the many motives which draw scientists to this branch of science (questions of the formation of stars and the cosmos, of the development of generators to produce electric energy directly from thermal energy, of the construction of magnetohydrodynamic pumps, and many other things).

On the other hand, one might point out the development of another special question. This relates to the numerous papers, begun in the USSR by Drabkina [1], and devoted to study of shock waves in air, generated by the electrical discharge of capacitors.

As regards the wider class of hydrodynamic phenomena (in electrically insulating liquids, that is) connected with, and at times even caused by the presence of large differences of potential, and where the role of the magnetic fields of the flowing currents is negligibly small, this branch of science is only in the initial stage of its development. This conclusion stems from the fact that, in innumerable published papers in this area, there has accumulated too much inconsistent empiricism, and too many arbitrary particular assumptions and arguments, while clear, connected ideas have been missing.

A particular difficulty in drawing the appropriate problems to popular attention is the accumulated conservatism in the following regard. If we are concerned with electrically insulating fluids, all our attention is concentrated on their dielectric properties, inhomogeneity of distribution of these properties (dielectric constant), dipole moments of molecules, electrostriction, their temperature dependences, etc. If we are concerned with high-voltage discharges, attention is concentrated on the popular phenomena of electric vacuum technology: electron emission, impact electrification, leaders, streamers, etc.

No attention is directed to the weak electrical conduction of such fluids, possible inhomogeneity in its distribution, or the resulting space charges and body forces as causes of self-motion of the fluid in the electric field.

The present review is an attempt to systematize some experimental facts and ideas, and also to identify physical and mathematical questions and problems stemming from the present state of the problem. In doing this, the greatest consideration will be given, in accordance with Lomonosov's precept, to self-motion of the fluid in the electric field, i.e., to free electric convection.

2. History of the question. If the form of convection that has been studied in greatest detail is gravitational-thermal, and that least studied—electromagnetic (pressure of light on gases, Lebedev's research), and if thermal-magnetic convection (Ageikin [2]) occupies an intermediate position, then electrical-conductive convection occupies a special position.

The phenomenon of free electric convection has been known for more than two hundred years: the electric wind, Franklin's wheel. The investigation of Arrhenius [3] could have given some quantitative eluc-

Table 1

Dimensionless Variables and Parameter of the Problem when Joule Heating is Important

Parameter	Unit	Determined from equations	Notation of dimensionless quantity
Length	$L_0 = \frac{\varepsilon}{4\pi\sigma_0}\left(\frac{cT_0\varkappa}{\nu}\right)^{1/2}$	(7)	$\frac{x}{L_0} = \xi, \ldots$
Electrical conductivity	σ_0	(5)	$\frac{\sigma}{\sigma_0} = s$
Temperature	T_0	(5)	$\frac{T}{T_0} = \theta$
Velocity	$v_0 = \left(\frac{cT_0\varkappa}{\nu}\right)^{1/2}$	(3), (7)	$\frac{\vec{v}}{v_0} = \vec{u}$
Electric field intensity	$E_0 = \frac{4\pi}{\varepsilon}(\mu\sigma_0)^{1/2}$	(3), (6)	$\frac{\vec{E}}{E_0} = \eta$
Potential	$\varphi_0 = \left(\frac{cT_0\gamma\varkappa}{\sigma_0}\right)^{1/2}$	(1), (3), (6), (7)	$\frac{\varphi}{\varphi_0} = \psi$
Measure of influence of convection compared with heat conduction	$A = \frac{\varepsilon cT_0}{4\pi\sigma_0\nu}$	(7)	A

Table 2

Dimensionless Variables when Joule Heating is not Important

Parameter	Unit	Determined from equations	Notation of dimensionless quantity
Characteristic dimension of equipment	L_0	—	$\frac{x}{L_0} = \xi, \ldots$
Velocity	$v_0 = \frac{4\pi\sigma L_0}{\varepsilon}$	(3)	$\frac{\vec{v}}{v_0} = \vec{u}$
Electric charge distribution density	$\rho_0 = \frac{(\mu\sigma)^{1/2}}{L_0}$	(2), (3), (6)	$\frac{\rho}{\rho_0} = W$
Electric field intensity	$E_0 = \frac{4\pi}{\varepsilon}(\mu\sigma)^{1/2}$	(3), (6)	$\frac{\vec{E}}{E_0} = \vec{\eta}$

idation of the hydrodynamics of this phenomenon, related to coronal discharge from a sharp point. He measured the forces acting on a point from which an electric wind was issuing, at different pressures in different gases. In this work, however, no mention was made of the calibration of the dynamometer which made the investigation possible.

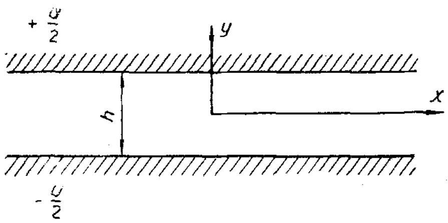

Fig. 1. Scheme for the plane problem.

Günterschulze and his colleagues [4,5], and Teichmann [6] investigated the rotary self-motion of a wire producing a corona discharge in air (the "corona motor"). Attempts to attribute the self-motion of the air around the heated and electrified wire to dielectric causes [7] were not successful: the electric conduction of the air in the form of a corona discharge [8] played a decisive role. Some investigators, even subsequently, did not avoid dielectric drag [9].

Quinke [10], Heidweiler [11], and Grätz [12] investigated torsional self-motion of solid cylinders and spheres between the plates of a plane capacitor filled with an insulating liquid, including self-excitation of torsional self-oscillations. Boltzmann [13] gave the correct explanation of these phenomena: the difference in the electrical conductivities of the solids and liquids used and the accumulation of surface charges.

A design for a kilovoltmeter [14] was proposed, based on the excitation in the transformer oil of an electric "wind" from a wire heated by the current, the "wind" cooling the wire and causing an imbalance in a bridge, this also serving as a measure of the applied voltage.

Avsec [15] published a number of papers devoted to various kinds of electric-convective motions in oil and in air. He criticized the neglectful attitude of investigators toward the hydrodynamics of the phenomena, but confined himself to a description of very interesting and numerous observations.

Arabadzhi [16] cursorily investigated the hydrodynamics of the electric wind in air, and in an electrically insulating liquid, experimentally.

An interesting example of the use of self-motion of a gas in an electric field is described in a paper by Foster et al. [17], who proposed a special vacuum pump. In it a heated cathode emits electrons which are attracted to a perforated anode. The electrons entrain molecules of the gas, and pump it into a cavity behind the anode. The converse idea—a dc ion generator—had been put forward earlier by Babat [18].

Mathematical theories of electrical and magnetic phenomena in moving bodies have been discussed by Eichenval'd [19]. At present, for slow moving media (subrelativistic velocity) the equation for the total current, presented below as (3), is used [20].

3. Electric convection when Joule heating is important. The phenomenon of self-motion of fluids in an electric field may be divided into two parts. In one class local Joule heating of the fluid by the flowing current plays an important role. The result is a local increase in the electrical conductivity of the fluid. In an electric field the liquid loses its electrical neutrality, and local space charges are created within it. The charged regions are subjected to the influence of Coulomb body forces, which also cause self-motion of the liquid.

This kind of motion may be attenuated, and it is then difficult to observe. It may be unattenuated (the electric "wind" from a sharp point in an insulating liquid). It may even be increasing (pre-breakdown phenomena in electrolytes).

This class of phenomena is described in the steady case by the full system of relevant equations of universal applicability:

$$\vec{E} = -\operatorname{grad}\varphi; \tag{1}$$

$$\operatorname{div}\vec{E} = \frac{4\pi}{\varepsilon}\rho; \tag{2}$$

$$\vec{J} = \sigma\vec{E} + \frac{\varepsilon}{4\pi}\vec{v}\operatorname{div}\vec{E} + \operatorname{rot}[(\vec{D}-\vec{E})\cdot\vec{v}]; \tag{3}$$

$$\operatorname{div}\vec{J} = 0; \tag{4}$$

$$\ln(\sigma/\sigma_0) = -T_0/T; \tag{5}$$

$$-\operatorname{grad}p + \mu\Delta\vec{v} + \rho\vec{E} = 0; \tag{6}$$

$$(\vec{v}\cdot\operatorname{grad}T) = \varkappa\Delta T + \sigma E^2/\gamma c; \tag{7}$$

$$\operatorname{div}\vec{v} = 0. \tag{8}$$

Equation (1) determines the electric field intensity vector $\vec{E}$ in terms of the potential φ; (2) relates the specific space charge ρ with $\vec{E}$, the permittivity ε being considered constant; (3) defines the component parts of the electric current density vector: the conduction current, the convection current, and the convective component of displacement current. Equation (4) reflects the law of conservation of electric charge, while (5) gives the most widespread dependence, even in many cases of rare precision, of the specific electrical conductivity σ of the substance on temperature T (the Boltzmann—Arrhenius—Frenkel rule). Here $T_0 = U_0/k$ denotes the "activation temperature" in analogy with the "activation energy" U_0; k denotes the Boltzmann constant. The value of T_0 is roughly 1200°–1800° K for aqueous electrolyte solutions, 5000° K for distilled water, and 10 000° K for high grade transformer oil. Equation (6) is a Navier-Stokes hydrodynamic equation (law of conservation of momentum) for steady laminar motion, and the motive force $\rho\vec{E}$ is Coulomb in nature; (7) is the energy equation (the Fourier-Kirchhoff heat equation, the heat source being the joule term σE^2, the mechanical equivalent of heat being taken into account in the volume heat capacity γC and in the thermal diffusivity $\varkappa$); (8) reflects the incompressibility of the liquid. Gravitational thermal convection is not taken into consideration.

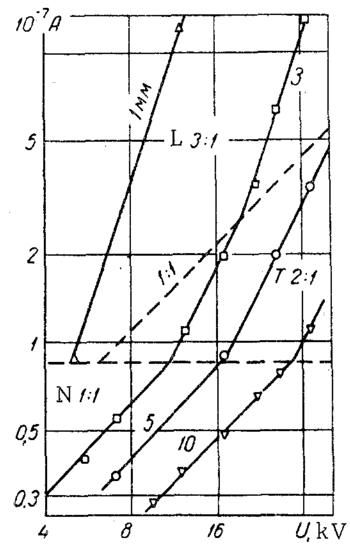

Fig. 2. Result of transforming Fig. 10 of [31] to logarithmic coordinates. The figures on the curves give the distances between the plane electrodes in mm. The horizontal dashed line at the level $0.85 \cdot 10^{-8}$ A corresponds to current density $1.73 \cdot 10^{-9}$ A/ /cm^2. Below this line is located the region N 1:1, corresponding to motionless transformer oil, in which the curves go along with slope 1:1 in accordance with Ohm's law. Above and to the left we have region L 3:1, corresponding to oil in laminar self-motion, the curves going along with slope 3 : 1 according to Eq. (17). To the right in the middle is located region T 2:1, corresponding to oil in turbulent self-motion, the curves proceeding with slope 2:1. It is as if the two last regions are separated one from the other by a line (dashed) with slope 1:1.

As a result of excluding the unknowns j, ρ, and p, we may combine (3) and (4), as well as (7) and (8). The new system of five equations will contain (1), (5), and (7), in addition to the following:

$$\sigma \operatorname{div} \vec{E} + \vec{E} \cdot \operatorname{grad} \sigma + \frac{\varepsilon}{4\pi} \vec{v} \cdot \Delta \vec{E} = 0; \qquad (9)$$

$$\operatorname{rot} \Delta \vec{v} = \frac{\varepsilon}{4\pi\mu} [\vec{E} \times \Delta \vec{E}]. \qquad (10)$$

The structure of the equations of the system determines the choice of natural units and of the dimensionless parameter of the problem in the manner shown in Table 1.

The data of the table permit the following system to be constructed in dimensionless variables:

$$\vec{\eta} = -\operatorname{grad} \psi; \qquad (11)$$

$$s \cdot \operatorname{div} \vec{\eta} + \vec{\eta} \cdot \operatorname{grad} s + \vec{u} \cdot \Delta \vec{\eta} = 0; \qquad (12)$$

$$\ln s = -1/\theta; \qquad (13)$$

$$\operatorname{rot} \Delta \vec{u} = [\vec{\eta} \times \Delta \vec{\eta}]; \qquad (14)$$

$$A(\vec{u} \cdot \operatorname{grad} \theta) = \Delta \theta + s\eta^2. \qquad (15)$$

This system describes a complex one-parameter nonlinear problem of high order (of derivatives). It is clear that its solution for any boundary conditions is a difficult problem.

We shall first suppose that the boundaries are isothermal, so that the only heating possible is that due to joule heating in the fluid itself. It can first be shown that the method of successive approximations may be used, say, in such a form. Having chosen appropriate boundary conditions, we must first solve the simplest electrostatic potential problem, which will simultaneously be a problem of direct current flow. This solution is the zero-order approximation. In the first approximation it is necessary to determine the temperature and the electrical conductivity in the motionless stream of liquid; in the second, the velocities; in the third, the correction to current density, etc. It then turns out that in the problem of the electric wind from a point—a paraboloid of revolution—the temperature distribution is a problem of unforeseeable complexity. The problem of a plane capacitor, for which the electrical conductivity of one of the plates varies periodically in space, gives an approximate variation of current density, after one cycle, which is less by several orders than in the zeroth approximation, i. e., the result is practically futile.

In the problem of the electric wind, a more promising method in practice is one in which the zeroth-order approximation would be already determined by the flow velocity, for example, in the shape of a laminar submerged jet [21], and at the same time the zero-order potential distribution as a near-paraboloid of revolution, etc. The remaining quantities would then be adjusted by calculation to the combined relations.

The following is noteworthy, in particular. Let the sharp point from which the electric wind flows into the liquid be idealized as a cylindrical rod with a hemispherical head of the same radius, located in a supposedly spherically symmetrical field in an inviscid liquid. Let it be maintained at potential U relative to an infinitely remote hemisphere. Then the resultant electric force acting on the head is independent of its radius, and is equal to

$$f = \frac{1}{4} \varepsilon U^2. \qquad (16)$$

This means that in water at a potential of only 2110 V a force of 1 g acts on a sharp point, reaching 1 kg at 67 kV. Extension of the head in the form of an elongated semiellipsoid of revolution decreases this force to 80% of the above values, in the limit (very sharp point). It is easy to imagine what kind of intense motion arises in the liquid, if the layer adjacent to the electrode, in virtue of its temporary electrical conductivity, possesses the properties of a fully movable electrode.

The phenomena described here should be distinguished from the similar phenomena of self-motion, in an electric field, of liquids which are heated, not by joule heat of a current flowing through them, but by heat from heated walls or electrodes [22]. The nonuniform heating of the liquid, closely related to nonuniformity in its electrical conductivity, leads, in an electric field, to excitation of space charges and body forces, and strong mixing of the liquid. This picture is observed very simply and clearly with the aid of half-shade instruments.

Neglect of the possibility of the above-mentioned spontaneous mixing of the liquid in these conditions led many investigators to far-reaching errors as regards the thermal conductivity of the liquid, this supposedly increasing sharply with the applied voltage (review [23]). In actual fact, the variations of molecular (but not convective or observed) thermal conductivity of liquids in an electric field may lie far below the sensitivity of the applied methods, and most likely are opposite in sign.

4. **Electrical convection when Joule heating is not important.** In a second class of phenomena joule heating is not important. It appears that these phenomena have not been observed in clear form. However, judging from their remote effects, they sometimes appear in a characteristic breakdown of Ohm's first law: The direct current through a plane capacitor filled with a weakly conducting dielectric is not proportional to the applied voltage.

For phemomena of the second class the system of equations is contracted and includes Eqs. (1)–(4), (6), and (8).

The structure of the new group of equations is determined by the choice of the natural units, shown in Table 2.

It is noteworthy that if we restrict ourselves to Eqs. (2), (6) and the second term on the right side of (3), the quantity of convection current density is expressed as

$$j_0 = (\varepsilon/4\pi)^2 (1/\mu) E_0^3 = (\varepsilon/4\pi)^2 (1/\mu) (U_0/L_0)^3. \qquad (17)$$

This means that for certain flows, which are similar one to another, the convection current density at all points in the stream will be proportional to the cube of the applied voltage. The convection current exceeds the conduction current, if the dimensionless voltage is large enough and exceeds a definite critical value.

We shall determine the conditions for which one class of phenomena may differ from the other [24]. To this end we shall give a detailed description of an expression for the body force—the cause of self-motion of the liquid:

$$\vec{f} = \rho \vec{E} = \frac{\varepsilon}{4\pi} \cdot \vec{E} \cdot \operatorname{div} \vec{E} = \frac{\varepsilon}{4\pi} \cdot \vec{E} \operatorname{div} \left(\frac{\vec{J}}{\sigma} \right) = \frac{\varepsilon}{4\pi} \cdot \vec{E} \left(\vec{J} \cdot \operatorname{grad} \frac{1}{\sigma} \right) = -\frac{\varepsilon}{4\pi} \cdot \vec{E} \left(\vec{J} \cdot \frac{\operatorname{grad} \sigma}{\sigma^2} \right). \quad (18)$$

In particular, in the one-dimensional case

$$f = \frac{\varepsilon}{4\pi} \cdot P \cdot \frac{d}{dx} \left(\frac{1}{\sigma} \right). \quad (19)$$

This formula shows that the role of joule heating, which is proportional to the specific power P generated by the current, is the less, the higher the resistivity $1/\sigma$, and the more rapid its variation in space. The interference from thermal-gravitational convection also increases with increase of heating. This means that phenomena of the second class are easily observed in highly insulating liquids of high quality and at high (prebreakdown) voltages.

5. Electric convection in a plane capacitor. The scheme for analytical solution of the simplest problem for the second class of phenomena is this. We shall examine the phenomena in an infinitely extended plane capacitor, filled with an incompressible liquid dielectric. We shall suppose that this dielectric is in a state of self-motion under the action of electrical forces, due to a difference in potential, U, between its plates. We shall confine ourselves to the case of the plane problem, where all the quantities are independent of coordinate z. The location of the axes of coordinates x and y is shown in Fig. 1. Combination of Eqs. (2)–(4) and (8), and taking the curl of (6) gives

$$\sigma\rho + (\vec{v} \cdot \operatorname{grad} \rho) = 0; \quad (20)$$

$$\mu \operatorname{rot} \Delta \vec{v} = [\operatorname{grad} \rho \times \operatorname{grad} \varphi]. \quad (21)$$

Introduction of the scalar stream function Φ frees the equations from the velocity vector,

$$v_x = -\frac{\partial \Phi}{\partial y}; \quad v_y = \frac{\partial \Phi}{\partial x}; \quad v_z = 0;$$
$$(\operatorname{rot} \vec{v})_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = \Delta\Phi. \quad (22)$$

Hence, taking (3) and (4) into account, we find

$$\frac{4\pi\rho}{\varepsilon} = -\Delta\varphi;$$
$$\frac{4\pi\sigma\rho}{\varepsilon} = \frac{\partial \rho}{\partial x} \cdot \frac{\partial \Phi}{\partial y} - \frac{\partial \rho}{\partial y} \cdot \frac{\partial \Phi}{\partial x};$$
$$\mu \Delta\Delta \Phi = \frac{\partial \rho}{\partial x} \cdot \frac{\partial \varphi}{\partial y} - \frac{\partial \rho}{\partial y} \cdot \frac{\partial \varphi}{\partial x}. \quad (23)$$

This system of three nonlinear scalar equations relative to three scalar functions of two space coordinates must be integrated, with boundary conditions at $\pm y = h/2$,

$$\varphi = \pm U/2; \quad \rho = 0; \quad \Phi = 0; \quad \partial \Phi / \partial y = 0. \quad (24)$$

These conditions denote that electrical double layers are not assumed at the electrodes, that they are impermeable to the liquid, and that there is an adhesive layer of liquid on them.

For further boundary conditions we shall confine ourselves to an investigation of only cellular flows, periodic in the x direction. A similar case has been examined in an application to thermal convection, by Rayleigh, Jeffries, Zirep [25–27], and others.

Let us transfer, for the sake of generality, to dimensionless variables with the aid of the following relations:

$$\xi = 2x/h; \quad \eta = 2y/h; \quad \psi = 2\varphi U;$$
$$W = 2\pi\rho h^2/\varepsilon U; \quad \Psi = \frac{\varepsilon}{4\pi\sigma} \left(\frac{2}{h} \right)^2 \Phi. \quad (25)$$

Then system (23) becomes

$$W = -\Delta\psi;$$
$$W = \frac{\partial W}{\partial \xi} \cdot \frac{\partial \Psi}{\partial \eta} - \frac{\partial W}{\partial \eta} \cdot \frac{\partial \Psi}{\partial \xi};$$
$$B^2 \Delta\Delta \Psi = \frac{\partial W}{\partial \xi} \cdot \frac{\partial \psi}{\partial \eta} - \frac{\partial W}{\partial \eta} \cdot \frac{\partial \psi}{\partial \xi}. \quad (26)$$

The dimensionless boundary conditions are written, when $\eta = \pm 1$, as

$$\psi = \pm 1; \quad \Psi = 0; \quad \frac{\partial \Psi}{\partial \eta} = 0; \quad W = 0. \quad (27)$$

In view of the assumed periodicity of the process with respect to ξ, and with the object of making the solution symmetrical with respect to the coordinate directions, we should seek a solution of the system in the form of functions expanded in doubly periodic Fourier series. In accordance with the example of Saltzmann [28], relating to Bénard thermal convection, we must substitute these expansions in the equations of system (26), and seek recurrence relations between their coefficients. In the simplest case Saltzmann had to use machine computation.

System (26) has a trivial zero-order solution, and we must seek a nontrivial solution. Each such solution is determined by its wave number with respect to ξ, and the existence of these solutions is determined by the parameter B of the problem, which is equal to the ratio of the mean field strength in the liquid to its natural unit according to Table 2. We must suppose that large applied voltage is required at very large and very small wave numbers. At some mean value or other of wave number a minimum critical voltage is required. In this aspect the problem comes into contact with the complex mathematical problem of bifurcation of solutions of systems of nonlinear partial differential equations.

As a result of the solution, we must find a value of the mean electric current density over the electrode surface in the adherent layer,

$$j^* = \frac{\sigma U}{h} \cdot \left[-\frac{\partial \Psi}{\partial \eta} \right]_{\eta=\pm 1} . \qquad (28)$$

In the trivial case it corresponds to Ohm's law. In the case of self-motion of a liquid we should expect large values of current density: in order to set the liquid in motion and to overcome viscous forces, additional power must be expended at the former voltage. Relation (17) may serve as a guide.

6. Experimental results. Because of the difficulties in solving (26), experimental investigations are of particular interest. An example is the work of Goodwin and Macfadyen [29], in which resistance measurements are presented for carefully purified normal hexane between plane electrodes (size not given), located at distances one from another of 55 to 240 μ. The results, presented in the form of a graph, show that the current increases strongly as the voltage increases. If we transform this graph to new variables, the cube root of the current as a function of the applied voltage, then the experimental points are located along rays which cut off roughly 500 V on the abscissa axis [30]. This shows that the applied voltage is expended mainly in creating in the liquid an electric field intensity whose cube turns out to be proportional to the current strength. It must therefore be assumed that the liquid, during the experiment, is in spontaneous laminar motion, given by relation (17). The remaining small part of the voltage may be attributed to overcoming the electric double layers at the electrodes or to removal of electrons from the electrodes. If we relate the slope of the rays in this picture to the interelectrode distance, it may be seen that, at any current, the voltage between the electrodes, as applied to the interior of the liquid, is proportional to the distance between electrodes. This means that we are not concerned here with unique boundaries or wall effects.

More detailed information is given in [31] on the electrical conductivity of transformer oil filling the space between circular plane-parallel electrodes of diameter 25 mm each, mounted at various distances one from another, in the range 1 to 10 mm. Figure 2 shows these results on a logarithmic scale. The complete diagram is divided into three regions, separated one from another by straight lines. In region N, corresponding to current density less than $1.73 \cdot 10^{-9}$ A/cm^2, Ohm's law is valid, the resistivity being $2.6 \cdot 10^{13} \pm 13\%$ ohm$\cdot$cm, and the limiting field strength about 37 kV/cm. It must be supposed that here the liquid remains motionless. The region corresponding to large currents at small interelectrode distances holds to a cubic dependence of current on voltage, in conformity with Eq. (17). It must be supposed that during the measurements the liquid here is in spontaneous laminar motion (region L).

At large voltages and large distances between electrodes, a region is wedged in, which is characterized by a quadratic dependence of current on voltage. It may be shown [32] by a qualitative calculation, similar to that leading to Eq. (17), that in turbulent self-motion of the liquid a quadratic dependence of current on voltage is obtained. It must be supposed that during the measurements, the liquid in this region is in spontaneous turbulent motion (region T).

Estimating the viscosity of transformer oil at 0.2 poise, with dielectric constant of 2.25, we obtain the value of the field strength element from Fig. 2 as

$$E_0 = 0.475\, CGSE = 142 \text{ V/cm}.$$

It then turns out that the critical value of dimensionless voltage is

$$B_{cr} = 37\,000/142 = 260.$$

Incidentally, the scatter of 13% shown in the resistivity values is made up of such concrete numbers which increase as the interelectrode distance decreases, that it may be interpreted as resulting from the action of the electric double layer near the electrodes at voltage about 1.8 kV.

From these examples, as well as certain others (e.g., [33]), it may be concluded that the application of the theory of similarity, as described, to (17), in particular, has been verified experimentally in its remote effects.

7. Probable role of hydrodynamic phenomena during electrical breakdown of liquids. An important example of electric discharges (of a charged capacitor) in liquids is breakdown in the form of an intense spark. Such a spark arises, not immediately following application of a voltage pulse, but after the lapse of a time ranging from small fractions of a microsecond to hundreds of microseconds. This is evidence of the existence of definte "prebreakdown" phenomena, which are sometimes so drawn out that the discharge is completed without breakdown (at large interelectrode distances). An investigation [34] of prebreakdown phenomena has shown that at that time branches of a brush-shaped gaseous cavity are created from the pointed electrode (or the point on it). The bases of these branches, adjacent to the point, sometimes emit light, mainly hydrogen lines in a background of continuous spectra.

To explain prebreakdown phenomena numerous theories have been proposed; these fall mainly into the following three groups: electric (or electronic), thermal, and bubble-type (cavitational). These theories do not satisfactorily agree with the experimental facts. All the authors at once reject assumptions about any kind of role of hydrodynamic processes. They cite the large inertia of the liquids, which "cannot" be set into appreciable motion in the space of microseconds.

Nevertheless, simple calculations on the basis of a primitive model (a combination of identical electrical and hydrodynamic potential fields) lead to the conclusion that in water, for example, at a voltage field of the order of 17 kV/cm, spontaneous hydrodynamic disturbances may arise, with diameter of tens of microns, and rate of growth by a factor e per microsecond. Although this problem requires further study, it is clear that hydrodynamic theories (in combination with the thermal and bubble theories) should not be left outside the attention of the investigators.

8. **What type of liquid?** We should finally outline the physical picture of the medium which appears under the designation liquid in the group of problems being studied. In this medium electrical conduction is accomplished only by the following mechanisms: ion-electrolytic (ion radicals may act as ions), and colloidal-phoretic (e.g., cataphoresis). The electronic mechanism of conduction is accomplished only in liquid metals not included in the examination, and in isolated examples of semiconducting liquids. The role of the remaining charge carriers is neglected: α-particles, β-rays, positrons, charged fragments of nuclear disintegration, etc. Thus, independent of the specific liquid, we may classify the nature of its electrical conduction by the concept of electrolyte, by including into a broad perspective of ions, not only proper ions but also more or less stable groups which are electrically charged and enlarged to the dimensions of colloidal particles.

The role of such large ions may be seen, in particular (when they have relatively small charge and large volume), in the fact that they slowly form loosely bonded electric double layers of great thickness at the electrodes, in the presence of a large potential difference (reaching kilovolts, maybe).

One of the most interesting groups of liquids is formed by the technical electrically-insulating oils of mineral origin with molecular weight around 300. These are mainly mixtures of nonpolar hydrocarbons, contaminated by impurities of many kinds. We may describe such a mixture as a single-humped curve (similar to a resonance curve) of distribution of molecules according to atomic weight. The purer the oil, the narrower and higher the curve.

When the temperature is increased in such liquids, a cracking process occurs, this being a combination of subdivision of the molecular weights of components with their enlargement. This process is due to thermal breakaway of homological chains to coalescence of the fragments obtained into new combination with fragments of other molecules. The process may be represented graphically as a gradual spreading out of the above-mentioned curve of distribution according to molecular weights.

A thorough study has been made of the cracking, which for industrial purposes, is carried out at high temperatures (over 300° C to obtain high yield of low-molecular gas and liquid fractions. The process does proceed, however, even at room temperature, in a very slow manner (unsuitable for industrial needs). A possible structure of certain fragments may satisfy the scheme of an ion-radical. They may be responsible for the low electrical conductivity observed for these oils.

If we call the process of spontaneous change in technical oils aging, it may be ascertained that in the literature aging processes are referred, almost without discussion, to oxidation.

Under the action of an electric field in these liquids new phenomena occur which have also been studied in some detail with the object of industrial utilization [35]. However, the physical picture of these phenomena remains quite cloudy, but no experimental investigation of the electric wind should be excluded from a scientific program. A wind of this kind arises at each sharp protuberance of the electrodes, or at dust particles adhering to them [22]. The fact is that the hydrodynamic particles taking part in the electric wind repeatedly fall for an instant into a small increased temperature zone ("repetitive cracking"), when the electric field strengths in them are very large. In these little-studied conditions various transformations of molecules are possible: formation of ion-radicals, temporary or stable increase of electric conductivity due to formation of ions from ion-radicals, enlargement and subdivision of molecules, formation of new structures with new properties, and so on. In particular, in operation the impregnation of capacitor oils with hydrogen (which also leads later to breakdown) has been registered, as well as precipitation of waxy substances (of high molecular weight) on the cable sheath.

It is clear that these processes are possible even in very carefully purified, chemically determined liquids. These liquids are formed to some extent or other throughout the experiment.

We should note particularly the industrial process of "voltalization" of lubricating oils, involving bombardment of thin layers of oil on the grounded electrode by air ions from a glow discharge.

Similarly, in an organic liquid, thermal motion disrupts the molecules and leads to the formation of ion radicals, and again, water dissociates thermally into the ion radicals H^+ proton and OH^- hydroxyl, with activation temperature T_0 about 6900° K. Injection into the water of high-energy particles, as well as of electrons, during transmission of current, so increases the concentration of these radicals that designs have been proposed for the electrolytic extraction of hydrogen peroxide ($2OH \rightarrow H_2O_2$). It is probable that this combination of hydroxyl radicals is the first stage in a chain of secondary phenomena during electrolysis of water, right up to liberation of gaseous oxygen [36]. The dissolved layers, passing through dissociation, have an appreciable influence on the behavior of the radicals. It may be seen from this brief outline that the picture of current transmission in a liquid is quite complex and little investigated. Undoubtedly, one cause of the lack of completeness of our knowledge on this question is the neglect by investigators of possible spontaneous hydrodynamic streams, and the distortion, and far from satisfactory reproducibility of the results of measurements. It is therefore necessary to distinguish carefully phenomena occurring within a motionless medium, from the results of its hydrodynamic motion, even on a microscopically small scale. Determination of the electrical conductivity of a liquid should be carried out in a medium which is clearly at rest.

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